Preprints of the 20th World Congress
The International Federation of Automatic Control
Toulouse, France, July 9-14, 2017

Quasi full information feedback control law of linear systems ⋆⋆

Shuyou Yu [*,**] **Jing Wang** [**] **Jung-Su Kim** [***] **Hong Chen** [*]
Frank Allgöwer [****]

[*] *State Key Laboratory of Automotive Simulation and Control, Jilin University, Changchun China 130012 (e-mail: {shuyou,chenh}@jlu.edu.cn)*
[**] *Department of Control Science and Engineering, Jilin University, Changchun China 130012 (e-mail: jean-hardworking@163.com)*
[***] *Department of Electrical and Information Engineering, Seoul National University of Science and Technology, Seoul 139-743, Korea (e-mail: jungsu@seoultech.ac.kr)*
[****] *Institute for Systems Theory and Automatic Control, University of Stuttgart, Germany (e-mail: frank.allgower@ist.uni-stuttgart.de)*

Abstract: In this paper, disturbance observer based quasi full information feedback control law of linear systems is proposed, where the feedback control law is with direct measurement of the plant states and the estimation of the disturbances. It shows that the system under control is input to state stable from the derivative of the disturbances to the system states provided that the given matching condition is satisfied. Furthermore, the effect of the disturbances can be compensated entirely if the disturbance is constant. The proposed scheme is verified by a simple numerical example and the stability control of four-wheel steering vehicle.

Keywords: linear systems, robust control, disturbance observer, input-to-state stability, disturbance rejection

1. INTRODUCTION

The ubiquitous existence of disturbances or perturbations requires the consideration of robust control strategies. In general, there are two intuitive ways to deal with disturbances or perturbations of systems. One tries to attenuate the effects of the disturbances or perturbations, for example, loop-shaping, robust control (Doyle et al., 1992; Zhou et al., 1996). The other designs disturbance observers based controller (DOBC) or active disturbances rejection controllers (ADRC) to compensate the disturbances or perturbations directly (Han, 1995; Guo and Cao, 2013; Li et al., 2014; Chen et al., 2016). Compared with robust control, DOBC or ADRC can “reject” disturbances or perturbations without scarifying its nominal performance.

In general, the existing disturbance observer based controller are confined to uncertain systems which satisfy a matching condition (Gutman and Letmann, 1976; Barmish and Leitmann, 1982). The matching condition refers to the uncertainties or perturbations acting on the system via the same channel as control input. Although some schemes (Huang and Xue, 2014; Yang et al., 2011) claimed that mismatched disturbances or perturbations can be coped with, it either requires to transform the mismatched disturbances or perturbations into the matched one first (Chen et al., 2016) or counteracts the mismatched disturbances or perturbations from the output channel only.

In this paper disturbance observer based feedback control law of linear systems is proposed, in which not only state feedback but also disturbance feedback are involved. We name it as quasi full information feedback control since only the estimation of disturbances is adopted. We show that the error of observer is input to state stable from the derivative of disturbances to system states. In particular, the error is zero while the disturbance is constant or goes to constant after a while, no matter how big the magnitude of the disturbances is. It allows to attenuate or compensate the effect of disturbances if a matching condition is satisfied.

The remainder of the paper is organized as follows. Section 2 formulates the problem. The properties of disturbance observer and the properties of systems under control are provided in Section 3 and Section 4, respectively. Two simulation examples are given in Section 5 to demonstrate the effectiveness of the proposed scheme. The paper is concluded with a short summary.

2. PROBLEM SETUP

Consider the following linear time-invariant (LTI) systems

⋆ The first version of this paper was finished while Shuyou Yu was visiting Institute for Systems Theory and Automatic Control, University of Stuttgart, Germany.

⋆⋆Shuyou Yu, Jing Wang and Hong Chen would like to thank the National Natural Science Foundation of China for financial support within the projects No.61573165 and No.61034001. This research was partially supported by the National Research Foundation of Korea Grant funded by the Korean Government (NRF-2015R1D1A1A01060588).

$$\dot{x} = Ax + Bu + B_w w, \tag{1}$$

where $x \in R^{n_x}$ is the system state, $u \in R^{n_u}$ the control input, $w \in R^{n_w}$ the uncertainty. The term w refers to exogenous disturbances, and differences or errors between its model and reality.

For system (1), the following state-space disturbance observer (Chen et al., 2000; Chen, 2004; Yang et al., 2011) is designed

$$\begin{cases} \dot{p} = -LB_w(p + Lx) - L(Ax + Bu), \\ \hat{w} = p + Lx, \end{cases} \tag{2}$$

where $\hat{w}$ is an estimate of the disturbance, p is an auxiliary variable, and L is the observer gain to be designed. Assume that the system state is measurable.

A composite linear control law is proposed

$$u = Kx + K_w \hat{w} \tag{3}$$

where K is a state feedback control gain, K_w is the disturbance compensation gain. The aim of designing K_w is to eliminate or reduce the effect of the disturbances on the system. Note that the composite linear control law (3) is consisted of the information of system state x and the estimate of the error w. Thus, it is a quasi full information control law.

In order to guarantee stability of closed-loop systems, the following assumptions are made:

Assumption 1. The derivative of $w(t)$ is piecewise continuous and $\|\dot{w}(t)\| \leq \alpha$ for all $t \geq 0$, where $\alpha > 0$ is a constant.

Assumption 2. (A, B) is stabilizable.

3. PROPERTIES OF DISTURBANCE OBSERVERS

The ultimate bounded property of the disturbance observer (2) is concluded by the following theorem.

Theorem 1. Suppose that Assumption 1 and Assumption 2 are satisfied for system (1). Then, there exist $M \geq 1$ and $\beta < 0$ such that the disturbance estimate $\hat{w}(t)$ yielded by the disturbance observer (2) can asymptotically track the disturbances $w(t)$ with the ultimate bound error $-\frac{\alpha M}{\beta}$ if the observer gain L is chosen such that $-LB_w$ is Hurwitz.

Proof 1. *The disturbance estimation error of the disturbance observer* (2) *is defined as*

$$e = \hat{w} - w. \tag{4}$$

Combining system (1) *and disturbance observer* (2), *the dynamics of the disturbance estimation error is*

$$\begin{aligned} \dot{e} =& \dot{\hat{w}} - \dot{w} \\ =& \dot{p} + L\dot{x} - \dot{w} \\ =& - LB_w\hat{w} - L(Ax + Bu) \\ &+ L(Ax + Bu + B_w w) - \dot{w} \\ =& - LB_w(\hat{w} - w) - \dot{w} \\ =& - LB_w e - \dot{w}. \end{aligned} \tag{5}$$

The solution of Eq.(5) *is*

$$e(t) = e^{-LB_w t}e(0) - \int_0^t e^{-LB_w(t-\tau)}\dot{w}d\tau. \tag{6}$$

Due to Assumption 1, we know that

$$\begin{aligned} \|e(t)\| &\leq \|e^{-LB_w t}e(0)\| + \left\| \int_0^t e^{-LB_w(t-\tau)}\dot{w}d\tau \right\| \\ &\leq \|e^{-LB_w t}e(0)\| + \alpha \int_0^t \|e^{-LB_w(t-\tau)}\|d\tau. \end{aligned} \tag{7}$$

As $-LB_w$ *is Hurwitz, there exist* $M \geq 1$ *and* $\beta < 0$ *(Blondel and Megretski, 2004) such that*

$$\|e^{-LB_w t}\| \leq Me^{\beta t}, \forall t \geq 0.$$

Then,

$$\begin{aligned} \|e(t)\| &\leq Me^{\beta t}\|e(0)\| + \alpha \int_0^t Me^{\beta(t-\tau)}d\tau \\ &= Me^{\beta t}\|e(0)\| - \frac{\alpha M}{\beta}\left(1 - e^{\beta t}\right) \\ &= M\left(\|e(0)\| + \frac{\alpha}{\beta}\right)e^{\beta t} - \frac{\alpha M}{\beta}. \end{aligned} \tag{8}$$

As $\beta < 0$, $\alpha > 0$ *and* $M \geq 1$, $\|e(t)\|$ *is bounded. Furthermore, as* $t \to \infty$,

$$\|e(t)\| \to -\frac{\alpha M}{\beta}. \tag{9}$$

This implies that the disturbance estimate of the disturbance observer can asymptotically track the disturbance with an ultimate bounded error of estimate.

Remark 1. Denote $\vartheta(t) := M\left(\|e(0)\| + \frac{\alpha}{\beta}\right)e^{\beta t} - \frac{\alpha M}{\beta}$. Note that $\vartheta(t)$ is monotonically decreasing with t. Thus, $\max_{t\geq 0}\|e(t)\| = M\|e(0)\|$ and $\min_{t\geq 0}\|e(t)\| = -\frac{\alpha M}{\beta}$.

Remark 2. Eq.(9) shows that the error of estimate as $t \to \infty$ is determined by $\|\dot{w}\|$ and the gain L.

Corollary 1. Suppose that Assumption 1 and Assumption 2 are satisfied for system (1), and $\lim_{t\to\infty}\dot{w}(t) = 0$. Then, the disturbance estimates $\hat{w}$ yielded by the disturbance observer (2) can asymptotically track the disturbance w without error if the observer gain matrix L is chosen such that $-LB_w$ is Hurwitz.

Proof 2. *In terms of Theorem 1,* $\|e(t)\|$ *is bounded for all* $t \geq 0$. *Without loss of generality, suppose that* $\|e(t)\| \leq S$ *for all* $t \geq 0$. *As* $\lim_{t\to\infty}\dot{w}(t) = 0$, *for a given* $\varepsilon > 0$, *there exists* T_0 *such that* $\|\dot{w}(t)\| \leq -\frac{\varepsilon\beta}{2M}$ *for all* $t \geq T_0$. *For simplification, denote* $h := -\frac{\varepsilon\beta}{2M}$.

The solution of Eq.(5) *is*

$$e(t) = e^{-LB_w(t-T_0)}e(T_0) - \int_{T_0}^t e^{-LB_w(t-\tau)}\dot{w}d\tau. \tag{10}$$

As $-LB_w$ *is Hurwitz and* $\|\dot{w}(t)\| \leq h$,

$$\begin{aligned} \|e(t)\| &\leq \|e^{-LB_w(t-T_0)}\|\|e(T_0)\| + h\int_{T_0}^t \|e^{-LB_w(t-\tau)}\|d\tau \\ &\leq Me^{\beta(t-T_0)}\|e(T_0)\| + hM\int_{T_0}^t e^{\beta(t-\tau)}d\tau \\ &\leq MSe^{\beta(t-T_0)} - \frac{hM}{\beta}\left(1 - e^{\beta(t-T_0)}\right) \\ &= -\frac{hM}{\beta} + \left(MS - \frac{hM}{\beta}\right)e^{\beta(t-T_0)} \end{aligned} \tag{11}$$

In terms of $h = -\frac{\varepsilon\beta}{2M}$, $-\frac{hM}{\beta} \leq \frac{\varepsilon}{2}$. *Furthermore, while* $t \geq T_0 + \frac{1}{\beta}\ln\left(\frac{\varepsilon}{2MS+\varepsilon}\right)$,

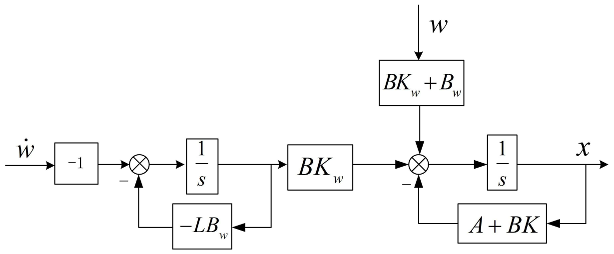

Figure 1. Diagram of system dynamics

$$\left(MS - \frac{hM}{\beta}\right) e^{\beta(t-T_0)} \le \frac{\varepsilon}{2}. \tag{12}$$

Thus, for any $\varepsilon > 0$, *there exists*

$$T := \max\left\{T_0, T_0 + \frac{1}{\beta}\ln\left(\frac{\varepsilon}{2MS+\varepsilon}\right)\right\}$$

such that for all $t \ge T$,

$$\|e(t)\| \le \varepsilon. \tag{13}$$

Therefore, $\lim_{t\to\infty} e(t) = 0$.

4. PROPERTIES OF SYSTEMS

Combining system (1), the quasi full information control law (3) and the dynamics of the disturbance estimation error (5), the closed-loop system is

$$\begin{bmatrix}\dot{x}\\ \dot{e}\end{bmatrix} = \begin{bmatrix}A+BK & BK_w\\ 0 & -LB_w\end{bmatrix}\begin{bmatrix}x\\ e\end{bmatrix} + \begin{bmatrix}BK_w+B_w & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}w\\ \dot{w}\end{bmatrix} \tag{14}$$

The diagram of system dynamics of the closed-loop systems Eq. (14) is referred to Fig.1, where w and $\dot{w}$ are looked upon as separated disturbance inputs.

A matching condition is required in order to guarantee the existence of K_w. Here the matching condition refers to that the disturbance/uncertainty enters a system via the same channels as control inputs, or the disturbances can be transformed into the same channels as the control inputs by the change of coordinate (Chen et al., 2016).

Assumption 3.

$$R(B_w) \subseteq R(B)$$

where $R(M)$ denotes the range (column) space of M.

In other words, $\operatorname{rank}[B] = \operatorname{rank}[B\ B_w]$. To the opinion of system theory, the assumption means that there exists at least a control input which stays in the same channel of a disturbance input, and tries to cancel out the effect of the disturbance input.

Theorem 2. Suppose that Assumption 1-3 are satisfied for system (1). If K, L and K_w are chosen such that

(1) both $-LB_w$ and $A+BK$ are Hurwitz,
(2) $BK_w + B_w = 0$.

Then,

(a) system (1) under the quasi full information feedback control law (3) is input-to-state stable (ISS) from $\dot{w}$ to state x,
(b) system state $x(t)$ is bounded which is proportional to $\|\dot{w}(t)\|_\infty$.

Proof 3. *(a) In terms of* $BK_w + B_w = 0$, *Eq.(14) can be rewritten as*

$$\begin{bmatrix}\dot{x}\\ \dot{e}\end{bmatrix} = \begin{bmatrix}A+BK & BK_w\\ 0 & -LB_w\end{bmatrix}\begin{bmatrix}x\\ e\end{bmatrix} + \begin{bmatrix}0\\ -1\end{bmatrix}\dot{w}. \tag{15}$$

Since both $-LB_w$ *and* $A+BK$ *are Hurwitz, the matrix*

$$\begin{bmatrix}A+BK & BK_w\\ 0 & -LB_w\end{bmatrix}$$

is Hurwitz as well. Thus, the closed-loop system is input-to-state stable (Khalil, 2002) from $\dot{w}$ *to state* x.

(b) Since the closed-loop system (14) is input-to-state stable, there exist $\mathcal{KL}$ *function* $\beta(.,.)$ *and* $\mathcal{K}$ *function* $\gamma(.)$ *such that*

$$\left\|\begin{bmatrix}x(t)\\ e(t)\end{bmatrix}\right\| \le \beta\left(\left\|\begin{bmatrix}x(0)\\ e(0)\end{bmatrix}\right\|, t\right) + \gamma\left(\sup_{0\le\tau\le t}\|\dot{w}\|\right), \qquad t \ge 0.$$

Furthermore, for $\|\dot{w}\| \le \alpha$, $\|x(t)\|$ *is bounded for all* $t \ge 0$ *(Khalil, 2002) and the bound is not related to the value of the amplitude of* $w(t)$.

The system (1) is ISS from $\dot{w}$ to x since the dependency of the state on w is eliminated by K_w and dependency of the state on $\dot{w}$ is made due to introduction of e for analysis purpose.

Remark 3. In terms of Theorem 2, system state $x(t)$ will converge to a disk centered at the equilibrium while t goes to infinity. The radius of the dish is determined by the gain matrices K, K_w and L, and the $\max_{t\ge0}\|\dot{w}(t)\|$, but not related to $\|w(t)\|$. That is, from standpoint of theory, Theorem 2 guarantees that system state $x(t)$ will converge to somewhere even if $\|w(t)\|$ is unbounded.

Corollary 2. Suppose that Assumption 1-3 are satisfied for system (1), and $\lim_{t\to\infty}\dot{w}(t) = 0$. If K, L and K_w are chosen such that

(1) both $-LB_w$ and $A+BK$ are Hurwitz,
(2) $BK_w + B_w = 0$.

Then, $\lim_{t\to\infty} x(t) = 0$, i.e., the effect of the disturbance is eliminated as t goes to infinity.

Proof 4. *Without loss of generality, assume that* $\|x(t)\| \le H$ *with* $H > 0$ *as* $x(t)$ *is bounded. Since A+BK is Hurwitz, there exist* $M_0 \ge 1$ *and* $\beta_0 < 0$ *such that*

$$\left\|e^{A+BK}\right\| \le M_0 e^{\beta_0 t}, \quad \forall t \ge 0.$$

Since $BK_w + B_w = 0$, *the state-space representation of the closed-loop system (14) can be rewritten as*

$$\begin{aligned}\dot{x} &= (A+BK)x + BK_w e,\\ \dot{e} &= -LB_w e - \dot{w}.\end{aligned} \tag{16}$$

Denote $h_0 := -\frac{\varepsilon\beta_0}{2M_0\|BK_w\|}$. *In terms of Corollary 1,* $\lim_{t\to\infty} e(t) = 0$. *Thus, for any* $\varepsilon > 0$, *there exists* T_0 *such that* $\|e(t)\| \le h_0$ *while* $t \ge T_0$. *Similar to the proof of Corollary 1, we have*

$$
\begin{aligned}
\|x(t)\| \le & \|e^{(A+BK)(t-T_0)x(T_0)}\| \\
& + \int_{T_0}^{t} \|e^{(A+BK)(t-\tau)} BK_w e\| d\tau \\
\le & M_0 e^{\beta_0 (t-T_0)} \|x(T_0)\| \\
& + h_0 \|BK_w\| M_0 \int_{T_0}^{t} \|e^{\beta_0 (t-\tau)}\| d\tau \\
\le & M_0 H e^{\beta_0 (t-T_0)} - \frac{h_0 \|BK_w\| M_0}{\beta_0} \left(1 - e^{\beta_0 (t-T_0)}\right) \\
= & - \frac{h_0 \|BK_w\| M_0}{\beta_0} \\
& + \left(M_0 H - \frac{h_0 \|BK_w\| M_0}{\beta_0}\right) e^{\beta_0 (t-T_0)}.
\end{aligned}
\tag{17}
$$

In terms of $h_0 = -\frac{\varepsilon \beta_0}{2M_0 \|BK_w\|}$, *we have*

$$
-\frac{h_0 \|BK_w\| M_0}{\beta_0} = \frac{\varepsilon}{2}. \tag{18}
$$

Furthermore, while $t \ge T_0 + \frac{1}{\beta_0} \ln\left(\frac{\varepsilon}{2M_0 H + \varepsilon}\right)$,

$$
\left(M_0 H - \frac{h_0 \|BK_w\| M_0}{\beta_0}\right) e^{\beta_0 (t-T_0)} \le \frac{\varepsilon}{2}. \tag{19}
$$

Therefore, for any given $\varepsilon > 0$, *there exists* $T = \max\left\{T_0, T_0 + \frac{1}{\beta_0} \ln\left(\frac{\varepsilon}{2M_0 H + \varepsilon}\right)\right\}$ *such that* $\|x(t)\| \le \varepsilon$ *while* $t \ge T_0$. *Thus,* $\lim_{t\to\infty} x(t) = 0$.

Remark 4. The condition $R(B_w) \subseteq R(B)$ in Theorem 2 and Corollary 1 indicates that the disturbance $w(t)$ has no direct influence on the system dynamics.

Remark 5. Due to Theorem 2, the disturbance $w(t)$ can be compensated completely via $\lim_{t\to\infty} \dot{w}(t) = 0$.

The proposed algorithm aims to guarantee robust stability of the closed-loop systems with respect to (decaying or non-decaying) disturbances. While Assumption 3 is satisfied, the steps to design a quasi full information state-feedback control law based on disturbance observers are as follows:

(1) Choose K_w such that $BK_w + B_w = 0$,
(2) Choose L such that $-LB_w$ is Hurwitz,
(3) Choose K such that $A + BK$ is Hurwitz,

i.e., the parameter (K_w, K, L) can be designed separately while Assumption 1-3 are satisfied.

Remark 6. The output in integral controllers of single-input single-output systems is directly proportional to the integral of the error signal. Thus, integral controller can return the controlled variable back to the exact set point following a disturbance. However, integral controller will change the order of the system, and has tendencies to make a system slower and the phase margin smaller. Adding integral control of a system may sacrifice the speed of the response, other performance measures, and even stability, for the sake of steady state error.

Instead, the proposed scheme works for multi-input multi-output systems, has no direct influence on the system dynamics and can be designed independently.

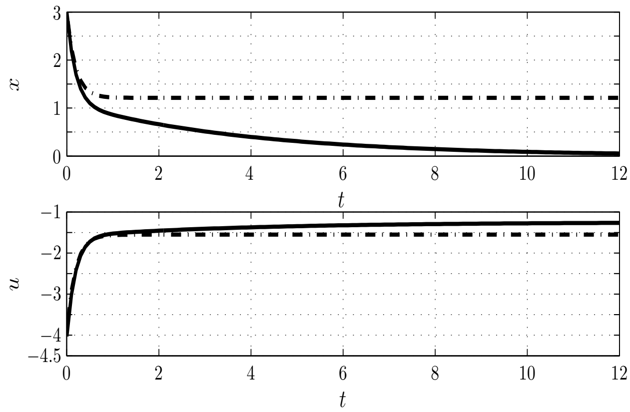

Figure 2. State and input trajectories for the initial state $x(0) = 3$ and $w(t) \equiv 0.5$. Solid line: with quasi full information control law, dashed line: with LQR control law.

5. SIMULATION RESULTS

In order to demonstrate the effectiveness of the proposed scheme, two examples are considered here. One is a numerical example, the other is the stability control of four-wheel steering vehicle.

5.1 Input perturbations

Consider the system described by

$$
\dot{x}(t) = x(t) + 4u(t) + w(t), \tag{20}
$$

which is an open-loop unstable linear system with $x(t)$, $u(t)$, $w(t) \in \mathcal{R}^1$. Assume that x can be measured instantaneously. The disturbance $w(t)$ is an unknown input perturbation.

Design a linear quadratic regulator (LQR) control law for the nominal system $\dot{x}(t) = x(t) + 4u(t)$ with weighting matrices $Q = 1$ and $R = 1$. The obtained linear state feedback control matrix and the Lyapunov matrix are $K = -1.2808$ and $P = 0.3202$, respectively.

For $B_w = 1$ and $B = 4$, the disturbance compensation gain is chosen as $K_w = -0.25$. Since the dynamics of the disturbance estimation error are $\dot{e} = -Le - \dot{w}$, choose $L = 0.25$ such that $-LB_w$ is Hurwitz and the control law is not aggressive.

The system trajectory starting from $x(0) = 3$ with LQR control law, and with quasi full information control law for system (20) are displayed in Fig.2, where the disturbance $w(t) \equiv 5$ for all $t \ge 0$. From Fig.2, we know that the system with LQR control law is ultimately bounded, and the system state will converge to the equilibrium while the proposed quasi full information control law is adopted.

Fig.3 shows that the system with the proposed scheme has the same dynamic as with LQR control law while the disturbance $w(t) \equiv 0$ for all $t \ge 0$, i.e., there is no disturbance at all.

5.2 Stability control of four-wheel steering vehicles

As a key technology of the advanced safety vehicle aiming to improve stability and maneuverability, stability control

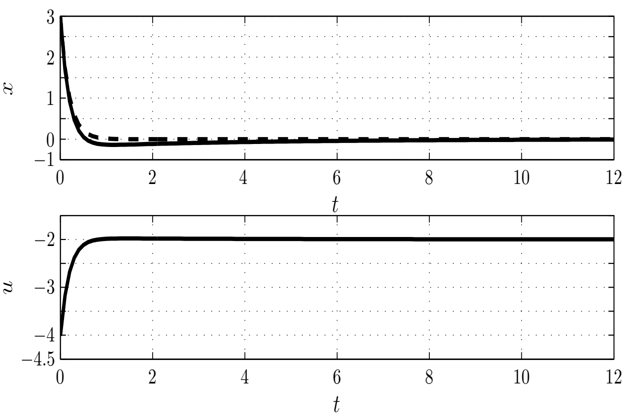

Figure 3. State and input trajectories for the initial state $x(0) = 3$ and $w(t) \equiv 0$. Solid line: with quasi full information control law, dashed line: with LQR control law.

of four-wheel steering vehicles have been studied intensively (Hiraoka et al., 2004; Wang et al., 2016). The purpose is to improve the control safety of four-wheel steering (4WS) vehicle with respect to cross-wind disturbances.

Assume that a vehicle with constant velocity moves with only two degree of freedom: lateral displacement and yaw rotation. The vehicle model considered is shown in Fig.??, where the body frame is fixed at the vehicle's center of gravity (CG).

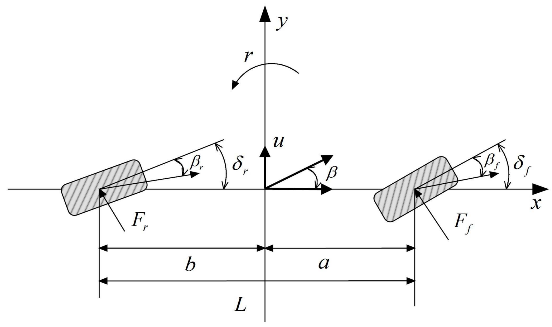

Figure 4. Diagram of 2 degree-of-freedom vehicle model

The following list describes the symbols used here.

β: sideslip angle
r: yaw rate of vehicle at CG
v: vehicle longitudinal velocity
m: vehicle mass
m_s: sprung mass
a: longitudinal distance from the front axle to CG
b: longitudinal distance from the rear axle to CG
l_w: the horizontal distance from the point crosswind force acting to CG
I_z: yaw inertia moment
k_φ: roll stiffness of vehicle suspension
k_f: cornering stiffness of front wheel
k_r: cornering stiffness of rear wheel
δ_f: the steering angles of the front wheels
δ_r: the steering angles of the rear wheels
F_f: the front lateral tyre force
F_r: the rear lateral tyre force

The key parameters of 4WS vehicle model used in this paper are as follows (Zhang, 2007): $m = 1500kg$, $m_s = 1300kg$, $I_z = 6000kg \cdot m^2$, $l_w = 0.2m$, $a = 1.1m$, $b = 1.4m$, $k_\varphi = 47250N/rad$, $k_f = 64000N/rad$, $k_r = 52000N/radr$.

Assume that β is small and v varies slowly, the front slip angle α_f and the rear slip angle α_r can be derived by

$$\begin{aligned} \alpha_f &= -\beta - \frac{ar}{v} + \delta_f, \\ \alpha_r &= -\beta + \frac{br}{v} + \delta_r. \end{aligned} \tag{21}$$

As shown in Fig.??, the coordinate frame is fixed at the vehicle's CG. In terms of Newton's second law, the vehicle dynamic equations can be written as

$$\begin{aligned} mv(\dot{\beta} + r) &= F_f + F_r, \\ I_z\dot{r} &= aF_f - bF_r. \end{aligned} \tag{22}$$

While the tyre slip angle is small, the front and rear lateral tyre forces vary linearly with their slip angles

$$\begin{aligned} F_f &= k_f\alpha_f, \\ F_r &= k_r\alpha_r. \end{aligned} \tag{23}$$

Denote $x = [\beta \ r]^T$ as the state vector, $u = [\delta_f \ \delta_r]^T$ the input vector. From (21) and (22), the state space model of the linear 4WS vehicle model is

$$\dot{x} = Ax + Bu, \tag{24}$$

where

$$A = \begin{bmatrix} -\frac{k_f+k_r}{mv} & \frac{bk_r-ak_f}{mv^2} - 1 \\ \frac{bk_r-ak_f}{I_z} & -\frac{a^2k_f+b^2k_r}{I_zv} \end{bmatrix}, \quad B = \begin{bmatrix} \frac{k_f}{mv} & \frac{k_r}{mv} \\ \frac{ak_f}{Iz} & -\frac{bk_r}{I_z} \end{bmatrix}.$$

Taking the exogenous disturbances caused by the cross-wind into account, Eq.(22) can be rewritten as

$$\begin{aligned} mv(\dot{\beta} + r) &= F_f + F_r + F_w, \\ I_z\dot{r} &= aF_f - bF_r + F_wl_w. \end{aligned} \tag{25}$$

The stationary wind force

$$F_w = \frac{1}{2}\rho C_F A_c v_w^2,$$

where ρ is the air density, A_c is the characteristic area of the vehicle, v_w is the speed of the wind and C_F is the non-dimensional aerodynamic coefficient which is usually a nonlinear function of the relative wind angle (Zhang, 2015). For simplification, in this paper, we choose $\rho = 1.2258kg/m^3$, $C_F = 0.7$ and $A_c = 0.001m^2$.

Thus, the related state space representation is

$$\dot{x} = Ax + Bu + B_ww, \tag{26}$$

where $w = \frac{F_w}{mv}$ is the normalized disturbance input, and $B_w = \begin{bmatrix} 1 & \frac{mvl_w}{I_z} \end{bmatrix}^T$.

The purpose is to design a control law such that the effect of the crosswind is attenuated or compensated while the vehicle is driving at a high speed. That is, the sideslip angle β and the yaw rate r of vehicle at CG are as small as possible with respect to the crosswind.

Since the matching condition $R(B_w) \subseteq R(B)$ is satisfied for the linear 4WS vehicle model, the quasi full information control law is designed. Firstly, a linear quadratic regulator control law is designed for the nominal system (24) with $Q = I_2$ and $R = 2 \times I_2$, where I_2 is 2-dimensional identity matrix. The obtained linear control law is $K =$

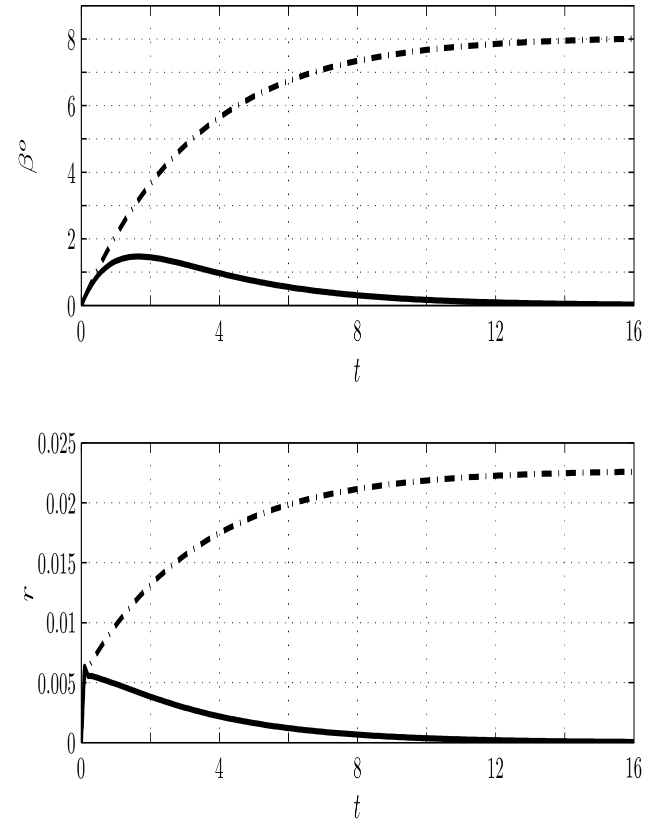

Figure 5. Comparison of side slip angle and yaw rate, dash-dotted line: with traditional LQR, solid line: with the proposed scheme.

$\begin{bmatrix} -0.0186 & 0.4751 \\ 0.2397 & -0.5067 \end{bmatrix}$. Choose the disturbance observer as $L = [0.5000\ 0.1667]$ which can guarantee that $-LB_w$ is Hurwitz. In order to compensate the effect of wind, choose $K_w = \begin{bmatrix} -0.9000 \\ -0.6231 \end{bmatrix}$ such that $BK_w + B_w = 0$.

Fig.4 compares sideslip angle and yaw rate at CG of the 4WS vehicle when driving with the speed of the vehicle $v = 30m/s$, the speed of the wind $v_w = 10m/s$ and the initial state $x(0) = [0\ 0]^T$. The sideslip angle of the vehicle with the traditional LQR is greater than 7^o sometime which causes the vehicle losing stability (van Zanten, 2000). However, the sideslip angle of the vehicle with the proposed control law stays in the interval $\beta \in [-2^o\ 2^o]$.

6. CONCLUSIONS

This paper was concerned with control of linear continuous-time systems with respect to exogenous disturbances. The control law is a composition of state and disturbance feedback controls, where the disturbances is estimated by a disturbance observer. Furthermore, the state feedback control law, observer gain and disturbance compensation gain can be designed respectively. It was shown that the influence of disturbances on system dynamics can be compensated if the matching condition is satisfied.

REFERENCES

Barmish, B. and Leitmann, G. (1982). On ultimate boundedness control of uncertain systems in the absence of matching assumptions. *IEEE Trans. Automat. Contr.*, 27(1), 153–158.

Blondel, V. and Megretski, A. (2004). *Unsolved Problems in Mathematical Systems and Control Theory.* Princeton University Press, Princeton, New Jersey.

Chen, W.H. (2004). Disturbance observer based control for nonlinear systems. *IEEE/ASME Transactions on Mechatronics*, 9(4), 706–710.

Chen, W.H., Yang, J., Guo, L., and Li, S. (2016). Disturbance-observer-based control and related methods - An overview. *IEEE Transactions on Industrial Electronics*, 63(2), 1083–1095.

Chen, W.H., Ballance, D.J., Gawthrop, P.J., and O'Reilly, J. (2000). A nonlinear disturbance observer for robotic manipulators. *IEEE Transactions on Industrial Electronics*, 47(4), 932–938.

Doyle, J.C., Francis, B.A., and Tannenbaum, A. (1992). *Feedback control theory.* Macmillan Publishing Comp., New York, NY.

Guo, L. and Cao, S. (eds.) (2013). *Anti-disturbance control for systems with multiple disturbances.* CRC Press, London, U.K.

Gutman, S. and Letmann, G. (1976). Stabilizing feedback control for dynamical systems with bounded uncertainties. In *Proc. IEEE Conf. Decision Contr.*, 94–99. IEEE.

Han, J. (1995). Extended state observer for a class of uncertain plants. *Control Decision*, 10(1), 85–88. In Chinese.

Hiraoka, T., Nishihara, O., and Kumamoto, H. (2004). Model-following sliding mode control for active four-wheel steering vehicle. *Review of Automotive Engineering*, 25, 305–313.

Huang, Y. and Xue, W. (2014). Active disturbance rejection control: Methodlogy and theoretical analysis. *ISA Trans.*, 53(4), 963–976.

Khalil, H.K. (2002). *Nonlinear Systems (third edition).* Prentice Hall, New York.

Li, S., Yang, J., Chen, W.H., and Chen, X. (eds.) (2014). *Disturbance observer-based control-Methods and applications.* CRC Press, London.

van Zanten, A. (2000). Bosch ESP systems: 5 years of experience. *SAE Technical Paper 2000-01-1633, doi:10.4271/2000-01-1633.*

Wang, J., Yu, S., and Chen, H. (2016). Model following control of active 4WS vehicle based on constrained output feedback H_∞ control. *Information and Control (In Chinese)*, 45(1), 53–59.

Yang, J., Zolotas, A., Chen, W.H., Michail, K., and Li, S. (2011). Robust control of nonlinear MAGLEV suspension system with mismatched uncertainties via DOBC approach. *ISA Transactions*, 50(3), 389–396.

Zhang, H. (2007). The research on stability control of high speed vehicle under crosswind based on active 4WS system. Ph.D. Thesis, Shanghai Jiao Tong University.

Zhang, X. (2015). Crosswind stability of vehicles under nonstationary wind excitation. Ph.D. Thesis, Karlsruher Institute for Technologie.

Zhou, K., Doyle, J.C., and Glover, K. (1996). *Robust and Optimal Control.* Prentice Hall, Upper Saddle River, New Jersey.